Problem sheet 8

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Solutions will be collected during the lecture on Monday December 16.

1. **[3 points]** Let u is a harmonic function. For which twice continuously differentiable function $f : \mathbb{R} \to \mathbb{R}$ the function $f(u)$ is also harmonic?

2. **[3+4 points]** In the following situations, find a holomorphic function f whose real part is u.

   (a) $u = x^2 - y^2 + y$;

   (b) $u = x^2 - y^2 + 5x + y - \frac{y}{x^2+y^2}$;

3. **[2+3 points]** For which φ the following functions are harmonic:

   (a) $u = \varphi(xy)$;

   (b) $u = \varphi(x^2 + y^2)$.

4. **[3 points]** Show that the functions e^z, $\cos z$ and $\sin z$ are holomorphic in the whole complex plane and compute their derivatives.