# Problem sheet 7

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*Solutions will be collected during the lecture on Monday December 9.*

1. **[4x3 points]** Using Stokes' theorem to compute the following line integrals
   (a) $\int_\gamma ydx + zdy + xdz$, where $\gamma$ is the circle $x^2 + y^2 + z^2 = a^2$, $x + y + z = 0$ with counter clockwise orientation when viewed from the positive side of axes $x$;
   (b) $\int_\gamma xydx + yzdy + zxdz$, where $\gamma$ is the intersection of the cylinder $x^2 + y^2 = 1$ with the plane $x + y + z = 1$ with counter clockwise orientation when viewed above;
   (c) $\int_\gamma (z^2 - x^2)dx + (x^2 - y^2)dy + (y^2 - z^2)dz$, where $\gamma$ is the intersection of the half sphere $x^2 + y^2 + z^2 = 9$, $z \geq 0$, with the cone $x^2 + y^2 = z^2$ with counter clockwise orientation when viewed above;
   (d) $\int_\gamma z^2dy + x^2dz$, where $\gamma$ is the curve $y^2 + z^2 = 9$, $4x + 3z = 5$ oriented clockwise viewed form the point $(0, 0, 0)$.

2. **[2 points]** For which $a \in \mathbb{C}$ the following function is continuous at 0?

$$f(z) = \begin{cases} \frac{\operatorname{Re} z}{z} & \text{if } z \neq 0, \\ a & \text{if } z = 0. \end{cases}$$

3. **[2+3 points]** For which real numbers $a$ and $b$ the function $f$ is holomorphic:
   (a) $f(z) = x + ay + i(bx + cy)$, $z = x + iy$;
   (b) $f(z) = \cos x\,(\cosh y + a \sinh y) + i \sin x\,(\cosh y + b \sinh y)$, $z = x + iy$?

4. **[4 points]** Let $z = re^{i\varphi}$ and $f(z) = u(r, \varphi) + iv(r, \varphi)$. Obtain Cauchy-Riemann equations in polar coordinates.

5. **[2 points]** Prove that the function $f(z) = \overline{z}$ is not complex differentiable.