Problem sheet 3

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Solutions will be collected during the lecture on Monday November 11.

1. **[3 points]** Find all $\alpha \in \mathbb{R}$ for which the integral

$$\iint\limits_{x^2+y^2\leq 1} \frac{dxdy}{(x^2+y^2)^\alpha}$$

converges.

2. **[3 points]** Check if the following integral converges

$$\iint_{\mathbb{R}^2} \sin(x^2+y^2)dxdy.$$

3. **[4 points]** Compute the integral

$$\iint_{\mathbb{R}^2} \frac{|x|dxdy}{(1+x^2+y^2)^2}.$$

4. **[4 points]** Let the curve γ is given by $\rho = \rho(\varphi)$, $\alpha \leq \varphi \leq \beta$, in polar coordinates. Prove that the length of γ equals

$$l(\gamma) = \int_\alpha^\beta \sqrt{\rho^2(\varphi) + \dot{\rho}^2(\varphi)}d\varphi.$$

5. **[3+4 points]** Find the length of the curves given by

   (a) $x = a\cos t$, $y = a\sin t$, $z = bt$, $t \in [0, 2\pi]$, where $a, b > 0$;

   (b) $\rho = a\varphi$, $0 \leq \varphi \leq 2\pi$ (in polar coordinates).

6. **[3 points]** Find a natural parametrisation of the cycloid $\gamma(t) = (a(t - \sin t), a(1 - \cos t))$, $t \in [0, 2\pi]$, where $a > 0$.