Problem sheet 2

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Solutions will be collected during the lecture on Monday November 4.

1. **[2+2 points]** Change the order of integrations in the following integrals

$$a)\quad \int_0^2 \left(\int_x^{2x} f(x,y)dy\right) dx; \quad b)\quad \int_1^e \left(\int_0^{\ln x} f(x,y)dy\right) dx.$$

2. **[2+3+3 points]** Evaluate the following integrals
   (a) $\iint\limits_S xy^2 dxdy$, where S is bounded by the parabola $y^2 = 4x$ and the line $x = 1$;
   (b) $\iint\limits_S (x^2+y^2)dxdy$, where S is the parallelogram bounded by the lines $y = x$, $y = x+a$, $y = a$ and $y = 3a$ $(a > 0)$;
   (c) $\iiint\limits_S (xy)^2 dxdydz$, where S is given by the inequalities $0 \leq x \leq y \leq z \leq 1$.

3. **[3+3 points]** Compute the following integrals
   (a) $\iint_S \sin\sqrt{x^2+y^2}dxdy$, where $S = \{(x,y):\ \pi^2 \leq x^2+y^2 \leq 4\pi^2\}$;
   (b) $\iiint_S (x^2+y^2)dxdydz$, where $S = \left\{(x,y,z):\ \frac{x^2+y^2}{2} \leq z \leq 2\right\}$.

4. **[4 points]** Compute the volume bounded by the surfaces $x^2+y^2+z^2 = 2az$, $x^2+y^2 \leq z^2$ $(a > 0)$.

5. **[3 points]** Let

$$B = \left\{x \in \mathbb{R}^d :\ \sum_{k=1}^d x_k^2 \leq 1\right\}$$

be the unit ball in $\mathbb{R}^d$ and

$$C = \left\{x \in \mathbb{R}^d :\ \sum_{k=1}^d \frac{x_k^2}{a_k^2} \leq 1\right\}$$

be the d-dimensional ellipsoid ($a_k > 0$, $k = 1,\dots,d$). Prove that the volume $\mu(C)$ of C equals $a_1 \dots a_d \mu(B)$.