Problem sheet 13
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Solutions will be collected during the lecture on Thursday January 30.

Points for solved exercises have to be included as bonus points for the homework

1. [**3 points**] Find a solution to the transport equation

$$2u_t(t,x) + x^3 u_x(t,x) = 0, \quad x \in \mathbb{R}, \quad t > 0,$$
$$u(0,x) = \sin x, \quad x \in \mathbb{R}.$$

2. [**3+6 points**] Solve the following heat equations:

   (a)

   $$u_t(t,x) = \frac{1}{2}u_{xx}(t,x) + x, \quad x \in \mathbb{R}, \quad t > 0,$$
   $$u(0,x) = 1, \quad x \in \mathbb{R};$$

   (b)

   $$u_t(t,x) = u_{xx}(t,x) + t, \quad 0 < x < 1, \quad t > 0,$$
   $$u(t,0) = 0, \quad u(t,1) = 0, \quad t \geq 0,$$
   $$u(0,x) = 0, \quad t \geq 0;$$

3. [**3+6 points**] Solve the following wave equations:

   (a)

   $$u_{tt}(t,x) = u_{xx}(t,x), \quad x \in \mathbb{R}, \quad t > 0,$$
   $$u(0,x) = x, \quad u_t(0,x) = x^2, \quad x \in \mathbb{R}.$$

   (b)

   $$u_{tt}(t,x) = 4u_{xx}(t,x), \quad 0 < x < 1, \quad t > 0,$$
   $$u(t,0) = 0, \quad u(t,1) = 0, \quad t \geq 0,$$
   $$u(0,x) = 0, \quad u_t(0,x) = x(1-x), \quad 0 \leq x \leq 1.$$