Problem sheet 12

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Solutions will be collected during the lecture on Monday January 27.

1. **[3+2+3+3 points]** Evaluate residues of the following functions at all isolated singularities:
   (a) $\frac{1}{z^3-z^5}$;
   (b) $\frac{\sin 2z}{(z+1)^2}$;
   (c) $z^3 \cos \frac{1}{z-2}$;
   (d) $\sin \frac{z}{z+1}$.

2. **[2+2+3+4+4 points]** Use the residue theorem to evaluate the following complex line integrals:
   (a) $\int\limits_{|z-2|=\frac{1}{2}} \frac{zdz}{(z-1)(z-2)^2}$;
   (b) $\int\limits_{|z|=1} \sin \frac{1}{z} dz$;
   (c) $\frac{1}{2\pi i} \int\limits_{|z|=2} \sin^2 \frac{1}{z} dz$;
   (d) $\frac{1}{2\pi i} \int\limits_{|z|=1} z^n e^{\frac{2}{z}} dz$, where n is an integer number;
   (e) $\int\limits_{|z|=4} \frac{z^{11}dz}{(z^6+2)^2}$. (*Hint:* Compute via residue at infinity)