Problem sheet 9

Tutorials by Ikhwan Khalid <ikhwankhalid92@gmail.com> and Mahsa Sayyary <mahsa.sayyary@mis.mpg.de>.
Solutions will be collected during the lecture on Wednesday June 12.

1. **[3+3 points]** Reduce to canonical form the following quadratic forms on $\mathbb{R}^3$:
   a) $Q(x) = x_1^2 + x_2^2 + 3x_3^2 + 4x_1x_2 + 2x_1x_3 + 2x_2x_3$;
   b) $Q(x) = 2x_1x_2 - 6x_2x_3 + 2x_1x_3$.

   The form $Q = a_1y_1^2 + a_2y_2^2 + \cdots + a_ny_n^2$, where $y_1, y_2, \ldots, y_n$ are new unknowns, are called canonical

2. **[3 points]** Find a basis in which the quadratic form on $\mathbb{R}^2$

$$Q(x) = x_1^2 - 3x_2^2 - 2x_1x_2$$

   has a normal form.

   The form from Th. 14.5 is called normal

3. **[3 points]** For which value of λ is the following quadratic form on $\mathbb{R}^3$ positive definite?

$$Q(x) = 4x_1^2 + 2x_2^2 + \lambda x_3^2 + 4x_1x_2 + 2x_1x_3.$$

4. **[2+2+2 points]** Consider the following set D. What are the limit points of D? Is D open? Is it closed? Is it bounded?
   a) $D = \left\{\frac{1}{n} : \ n \in \mathbb{N}\right\} \cup \{0\}$ as a subset of $\mathbb{R}$;
   b) $D = [0,1)^2 = \left\{(x,y) \in \mathbb{R}^2 : \ x, y \in [0,1)\right\}$ as a subset of $\mathbb{R}^2$;
   c) $D = \left\{(x,y) \in \mathbb{R}^2 : \ x < y\right\}$ as a subset of $\mathbb{R}^2$.

5. **[3 points]** Let A_1, A_2 be closed sets in $\mathbb{R}^d$. Using the definition of close set show that $A_1 \cap A_2$ and $A_1 \cup A_2$ are close.

6. **[4 points]** Show that $\bar{\bar{A}} = \bar{A}$.

   (Here $\bar{A}$ denotes the closure of A.)