UNIVERSITÄT LEIPZIG

Problem sheet 4

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Solutions will be collected during the lecture on Monday May 6.

1. [**2 points**] Compute the sign of the following permutation

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 3 & 1 & 6 & 10 & 2 & 5 & 4 & 7 & 8 & 9 \end{pmatrix}.$$

2. [**1+1 points**] Compute

$$a) \quad \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 5 & 1 & 2 & 4 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 1 & 5 & 2 \end{pmatrix}, \qquad b) \quad \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 1 & 2 \end{pmatrix}^3.$$

3. [**2 points**] Write the following permutation as a composition of transpositions:

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 5 & 4 & 3 & 1 \end{pmatrix}.$$

4. [**2 points**] Show that any transposition is an odd permutation.

5. [**1+2+3+3 points**] Compute the following determinants:

$$a) \begin{vmatrix} -1 & 3 \\ 2 & 1 \end{vmatrix}; \quad b) \begin{vmatrix} 2 & -1 & 3 \\ 1 & -5 & -2 \\ 2 & 1 & 4 \end{vmatrix}; \quad c) \begin{vmatrix} 2 & -5 & 1 & 2 \\ -3 & 7 & -1 & 4 \\ 5 & -9 & 2 & 7 \\ 4 & -6 & 1 & 2 \end{vmatrix}; \quad d) \begin{vmatrix} 1 & 2 & 3 & \dots & n \\ -1 & 0 & 3 & \dots & n \\ -1 & -2 & 0 & \dots & n \\ \dots & \dots & \dots & \dots & \dots \\ -1 & -2 & -3 & \dots & 0 \end{vmatrix}.$$

6. [**3 points**] Obtain the following expression of the Vandermonde determinant

$$\begin{vmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ 1 & x_3 & x_3^2 \end{vmatrix} = (x_2 - x_1)(x_3 - x_1)(x_3 - x_2).$$

7. [**3 points**] Solve the following equation

$$\begin{vmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & 1-x & 1 & \dots & 1 \\ 1 & 1 & 2-x & \dots & 1 \\ \dots & \dots & \dots & \dots & \dots \\ 1 & 1 & 1 & \dots & n-x \end{vmatrix} = 0.$$

8. [**3 points**] Let f_{ij} be differentiable functions on $\mathbb{R}$ and let F be the $n \times n$-matrix with entries f_{ij}. Show that $\det F$ is differentiable on $\mathbb{R}$ and

$$(\det F)' = \sum_{i=1}^{n} \det \begin{pmatrix} f_{11} & \dots & f_{1n} \\ \dots & \dots & \dots \\ f_{i-1,1} & \dots & f_{i-1,n} \\ f'_{i1} & \dots & f'_{in} \\ f_{i+1,1} & \dots & f_{i+1,n} \\ \dots & \dots & \dots \\ f_{n1} & \dots & f_{nn} \end{pmatrix}.$$