Problem sheet 13

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Solutions will be collected during the lecture on Monday July 8.

1. [**4 points**] Find local extrema of $f(x, y, z) = 2x + 3y + z$ subject to $x^2 + 2y^2 + 3z^2 = 1$.

2. [**3 points**] A rectangle has perimeter p. Find its largest possible area.

3. [**5 points**] Find the minimum value of

$$f(x, y, z, w) = x^2 + 2y^2 + z^2 + w^2$$

subject to

$$\begin{aligned} x + y + z + 3w &= 1, \\ x + y + 2z + w &= 2. \end{aligned}$$

4. [**3 points**] Verify that the function $y = \tan\left(\frac{x^3}{3} + C\right)$ is a solution to the differential equation $y' = x^2(1 + y^2)$ for any choice of a constant C.

5. [**3+3 points**] Solve the initial value problems:
a) $y' = x \ln x, \quad y(1) = -\frac{1}{4}$;
b) $y'' = -x \sin x, \quad y(0) = 1,\ y'(0) = -3$.

6. [**4 points**] Verify that the function $y = x^2(1 + \ln x)$ is a solution to the initial value problem

$$y'' = \frac{3xy' - 4y}{x^2}, \quad y(e) = 2e^2, \quad y'(e) = 5e.$$