# Problem sheet 12

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*Solutions will be collected during the lecture on Wednesday July 3.*

1. [**2 points**] Let $f(x, y) = (2xy, e^x + y)$, $(x, y) \in \mathbb{R}^2$. Show that the function $f$ is invertible in a neighborhood of the point $(1, 1)$.

2. [**3 points**] Compute the partial derivatives of the function $z = z(x, y)$ defined by the equation $x + y + z = e^z$.

3. [**2+3 points**] Compute the second order derivatives of the following functions:
   a) $f(x, y) = \ln\sqrt{x^2 + y^2}$; b) $f(x, y, z) = xy + yz + zx$.

4. [**3 points**] Let $a \neq 0$ and $b$ be constants. Show that the function
$$u = \frac{1}{2a\sqrt{\pi t}} e^{-\frac{(x-b)^2}{4a^2 t}}$$
   solves the equation
$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}.$$

5. [**4 points**] Write the Taylor series of the function $f(x, y) = 2x^2 - xy - y^2 - 6x - 3y + 5$ at the point $x_0 = (1, -2)$.

6. [**3+3+3 points**] Find a local extrema of the following functions:
   a) $f(x, y) = (x + y)e^{-x^2-y^2}$; b) $f(x, y) = x^3 + y^3 - 3xy$; c) $f(x, y, z) = x^2 + y^2 + z^2 + 12xy + 2z$.