Problem sheet 8

Tutorials by Dr. Michael Schnurr <michael.schnurr@mis.mpg.de> and Ikhwan Khalid <ikhwankhalid92@gmail.com>.
Solutions will be collected during the lecture on Wednesday December 19.

1. **[1x4 points]** Using L'Hospital's Rule, show that
   a) $\lim\limits_{x\to 0} \frac{\ln(1+x)}{\sin x} = 1$; b) $\lim\limits_{x\to e} \frac{(\ln x)^{\alpha} - \left(\frac{x}{e}\right)^{\beta}}{x-e} = \frac{\alpha-\beta}{e}$, where α, β are some real numbers;
   c) $\lim\limits_{x\to +\infty} \frac{\ln x}{x^{\varepsilon}} = 0$ for all $\varepsilon > 0$; d) $\lim\limits_{x\to +0} (\ln(1+x))^x = 1$.

2. **[3x3 points]** Using L'Hospital's Rule, compute the following limits:
   a) $\lim\limits_{x\to 0} \frac{e^x - e^{\sin x}}{x - \sin x}$; b) $\lim\limits_{x\to 0+} \left(\frac{\ln(1+x)}{x}\right)^{\frac{1}{x}}$; c) $\lim\limits_{x\to 0} \frac{(1+x)^{\frac{1}{x}} - e}{x}$.

3. **[2x3 points]** Compute the n-th derivative of the following functions:
   a) $f(x) = 2^{x-1}$, $x \in \mathbb{R}$; b) $f(x) = \sqrt{2x-1}$, $x > \frac{1}{2}$; c) $(x^2 e^x)^{(n)}$, $x \in \mathbb{R}$.

   (*Hint:* Use the Leibniz Formula in c))

4. **[3 points]** Write Taylor's expansion of the function e^{2x-x^2}, $x \in \mathbb{R}$ at the point $x_0 = 0$ up to the term with x^5.

5. **[2x2 points]** Use Taylor's formula to compute the limits
   a) $\lim\limits_{x\to 0} \frac{\cos x - e^{-\frac{x^2}{2}}}{x^4}$; b) $\lim\limits_{x\to 0} \frac{e^x \sin x - x(1+x)}{x^3}$.