



Problem sheet 6

Tutorials by Dr. Michael Schnurr <michael.schnurr@mis.mpg.de> and Ikhwan Khalid <ikhwankhalid92@gmail.com>.
Solutions will be collected during the lecture on Wednesday December 5.

- [1+1+1 points]** Express through $f'(a)$ the following limits:
a) $\lim_{h \rightarrow 0} \frac{f(a+2h)-f(a)}{h}$; b) $\lim_{h \rightarrow 0} \frac{f(a+h)-f(a-h)}{h}$; c) $\lim_{n \rightarrow \infty} n (f(\frac{n+1}{n}a) - f(a))$.
- [2 points]** Using the definition of derivative, check that $(x|x|)' = 2|x|$, $x \in \mathbb{R}$.
- [3 points]** For the function $f(x) = |x^2 - x|$, $x \in \mathbb{R}$, compute $f'(x)$ for each $x \in \mathbb{R} \setminus \{0, 1\}$.
Compute left and right derivatives at points 0 and 1.
- [1+2 points]** Let

$$f(x) = \begin{cases} x^2, & x \leq 1, \\ ax + b, & x > 1. \end{cases}$$

For which $a, b \in \mathbb{R}$ the function f :

- is continuous on \mathbb{R} ; b) is differentiable on \mathbb{R} ? Compute also f' .
- [2x3 points]** Check whether the following functions are differentiable at 0. Justify your answer.
a) $f(x) = \begin{cases} \frac{\cos x - 1}{x}, & x \neq 0, \\ 0, & x = 0; \end{cases}$ b) $f(x) = \sqrt[5]{x^2}$, $x \in \mathbb{R}$; c) $f(x) = |\sin x|$, $x \in \mathbb{R}$.
 - [1x8 points]** Compute derivatives of the following functions:
a) $f(x) = x^2 \sin x$; b) $f(x) = e^{-\frac{x^2}{2}} \cos x$; c) $f(x) = \frac{x}{1+x^2}$; d) $f(x) = \frac{e^x + e^{-x}}{e^x - e^{-x}}$;
e) $f(x) = 2^{\tan(x^2-1)}$; f) $f(x) = \sin(\cos^2(\tan^3 x))$; g) $f(x) = \sqrt[3]{\frac{1+x^3}{1-x^3}}$;
h) $f(x) = e^{ax} \cdot \frac{a \sin bx - b \cos bx}{\sqrt{a^2 + b^2}}$, where a, b are some constants.