



### Problem sheet 3

Tutorials by Dr. Michael Schnurr <michael.schnurr@mis.mpg.de> and Ikhwan Khalid <ikhwankhalid92@gmail.com>.  
Solutions will be collected during the lecture on Wednesday November 14.

- [1+1+1 points]** For a sequence  $(a_n)_{n \geq 1}$  compute  $\lim_{n \rightarrow \infty} a_n$  and  $\overline{\lim}_{n \rightarrow \infty} a_n$ , if for all  $n \geq 1$   
a)  $a_n = 1 + \frac{1}{n}$ ; b)  $a_n = 1 + n \sin \frac{n\pi}{2}$ ; c)  $a_n = \frac{(-1)^n}{n} + \frac{1+(-1)^n}{2}$ .
- [3 points]** Show that  $a := \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \inf_{k \geq n} a_k$ , for the case  $a \in \mathbb{R}$ .  
(The equality also holds in the case  $a \in \{-\infty, +\infty\}$ ).
- [2 points]** Check that the sequence  $(a_n = \frac{\sin 1}{2^1} + \frac{\sin 2}{2^2} + \dots + \frac{\sin n}{2^n})_{n \geq 1}$  is a Cauchy sequence.
- [2+2+2 points]** Find the domain and the range of the following functions:  
a)  $f(x) = \frac{1}{(x+1)^2}$ ; b)  $f(x) = \sqrt{1-x^2}$ ; c)  $f(x) = \ln(1+x)$ .
- [2+2 points]** Find the formulas for the following implicitly defined functions. What are their domains?  
a)  $y = f(x)$  is the solution to the equation  $x^3y + 2y = 5$ ;  
b)  $y = f(x)$  is the largest solution to the equation  $y^2 = 3x^2 - 2xy$ .