Problem sheet 2

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Solutions will be collected during the lecture on Wednesday November 7.

1. [**1+1+1 points**] Using the definition of the limit show that
   a) $\lim\limits_{n\to\infty} \frac{n-1}{n} = 1$; b) $\lim\limits_{n\to\infty} n^2 = +\infty$; c) $\lim\limits_{n\to\infty} (-1)^n$ doe not exist.

2. [**3 points**] Assume that $a_n \to a$, $n \to \infty$, and $b_n \to b$, $n \to \infty$. Show that $\max\{a_n, b_n\} \to \max\{a, b\}$, $n \to \infty$.

3. [**2+2+2 points**] Compute the following limits:
   a) $\lim\limits_{n\to\infty} \frac{n^3-2n^2\cos n+n}{\sqrt{n}-3n^3+1}$; b) $\lim\limits_{n\to\infty} (\sqrt{n^2+n} - \sqrt{n})$; c) $\lim\limits_{n\to\infty} \sqrt[n]{n^2 2^n + 3^n}$.

4. [**3 points**] Let $(a_n)_{n\geq 1}$ be a bounded sequence and $b_n \to 0$, $n \geq \infty$. Prove that $a_n b_n \to 0$, $n \to \infty$.

5. [**3+3 points**] Using the monotonicity compute the following limits:
   a) $\lim\limits_{n\to\infty} \frac{n!}{2^{n^2}}$; b) $\lim\limits_{n\to\infty} \underbrace{\sqrt{2+\sqrt{2+\ldots+\sqrt{2+\sqrt{2}}}}}_{n \text{ square roots}}$.

6. [**2 points**] Show that $\lim\limits_{n\to\infty} n \ln\left(1+\frac{1}{n}\right) = 1$.

7. [**2 points**] Identify the set of subsequential limits of the sequence $\left(\sin\frac{2\pi n}{3}\right)_{n\geq 1}$.